

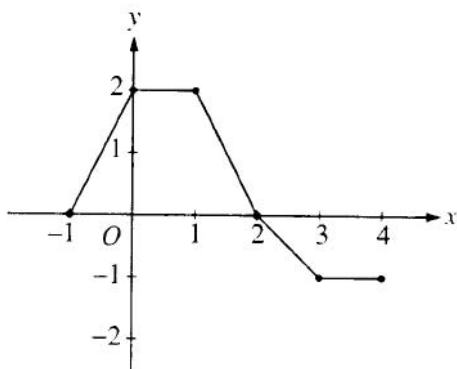
AP Calculus BC Mid-Term Review Problem Set

1.

What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

- (A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

2.



The graph of a piecewise-linear function f , for $-1 \leq x \leq 4$, is shown above. What is the value of

$$\int_{-1}^4 f(x) dx?$$

- (A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3.

$$\int_1^2 \frac{1}{x^2} dx =$$

- (A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

4.

If f is continuous for $a \leq x \leq b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \leq x \leq b$.

(D) f has a maximum value on $a \leq x \leq b$.

(E) $\int_a^b f(x) dx$ exists.

5.

$$\int_0^x \sin t \, dt =$$

(A) $\sin x$

(B) $-\cos x$

(C) $\cos x$

(D) $\cos x - 1$

(E) $1 - \cos x$

6.

If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$

(B) -2

(C) $\frac{2}{7}$

(D) $\frac{3}{2}$

(E) $\frac{7}{2}$

7.

$$\int_1^e \left(\frac{x^2 - 1}{x} \right) dx =$$

- (A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

8.

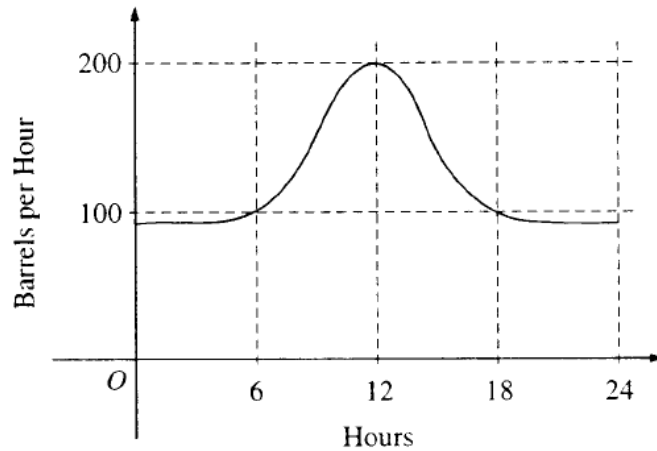
Let f and g be differentiable functions with the following properties:

- (i) $g(x) > 0$ for all x
- (ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

- (A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

9.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

- (A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

10.

What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

- (A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

11.

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) dx =$

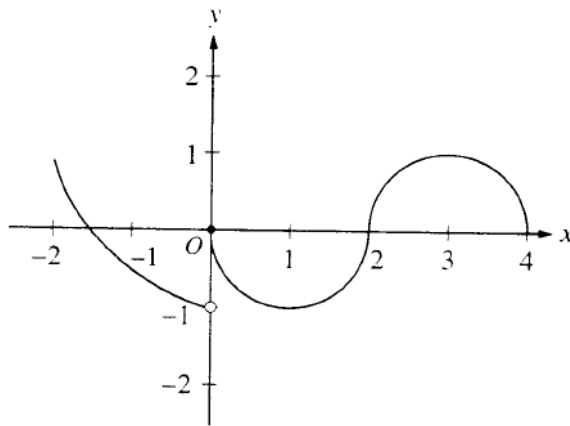
- (A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

12.

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then $\lim_{x \rightarrow 2} f(x)$ is

- (A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

13.



The graph of the function f shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x , $-2 < x < 4$, is f not differentiable?

- (A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14.

A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15.

If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

(A) -3

(B) -2

(C) 2

(D) 3

(E) 18

16.

If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $-\cos(e^{-x})$

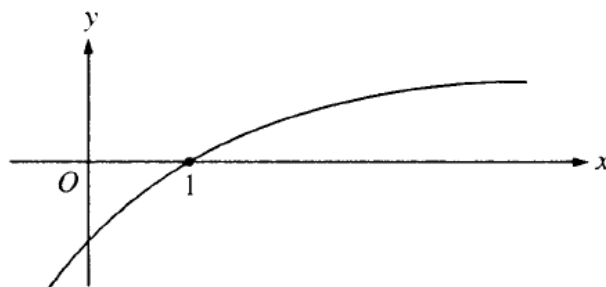
(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x} \cos(e^{-x})$

(E) $-e^{-x} \cos(e^{-x})$

17.



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

- (A) $f(1) < f'(1) < f''(1)$
- (B) $f(1) < f''(1) < f'(1)$
- (C) $f'(1) < f(1) < f''(1)$
- (D) $f''(1) < f(1) < f'(1)$
- (E) $f''(1) < f'(1) < f(1)$

18.

An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

- (A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

19.

If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

- (A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) $-1, 0,$ and 2 only

20.

What are all values of k for which $\int_{-3}^k x^2 dx = 0$?

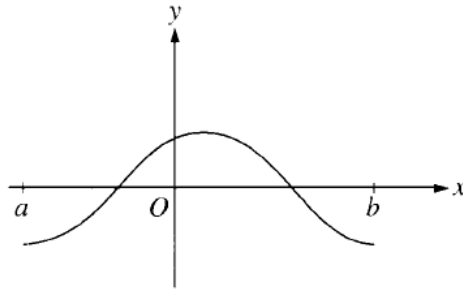
- (A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) $-3, 0,$ and 3

21.

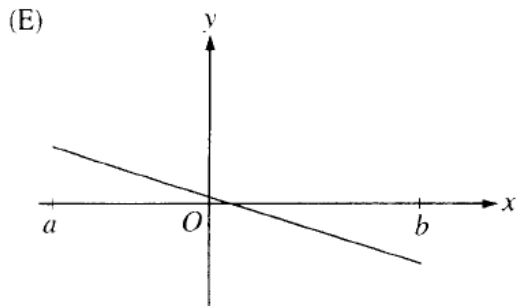
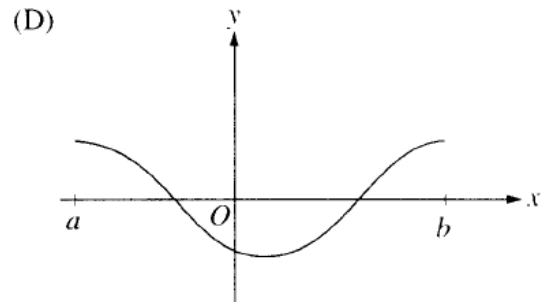
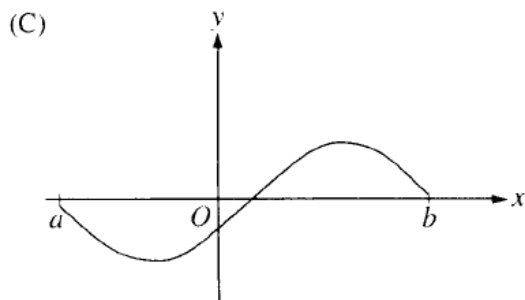
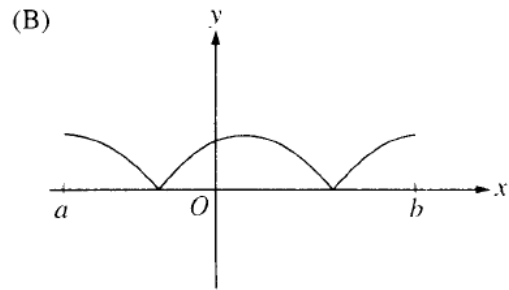
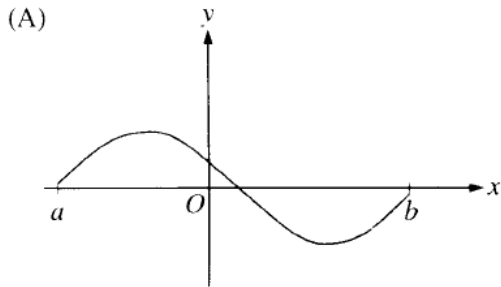
The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

- (A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
(C) $(0, \infty)$
(D) $(-\infty, 0)$
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

22.



The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f ?



23.

The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

- (A) 9 (B) 12 (C) 14 (D) 21 (E) 40

24.

What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

- (A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

25.

| | | | |
|--------|---|-----|---|
| x | 0 | 1 | 2 |
| $f(x)$ | 1 | k | 2 |

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

26.

What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

- (A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

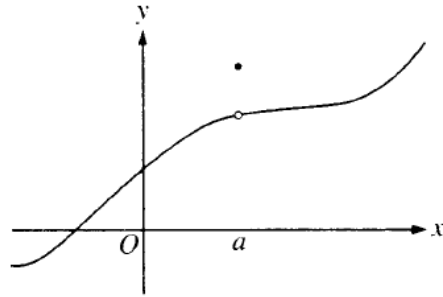
27.

If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

- (A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

Calculator

28.



The graph of a function f is shown above. Which of the following statements about f is false?

- (A) f is continuous at $x = a$.
- (B) f has a relative maximum at $x = a$.
- (C) $x = a$ is in the domain of f .
- (D) $\lim_{x \rightarrow a^+} f(x)$ is equal to $\lim_{x \rightarrow a^-} f(x)$.
- (E) $\lim_{x \rightarrow a} f(x)$ exists.

29.

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

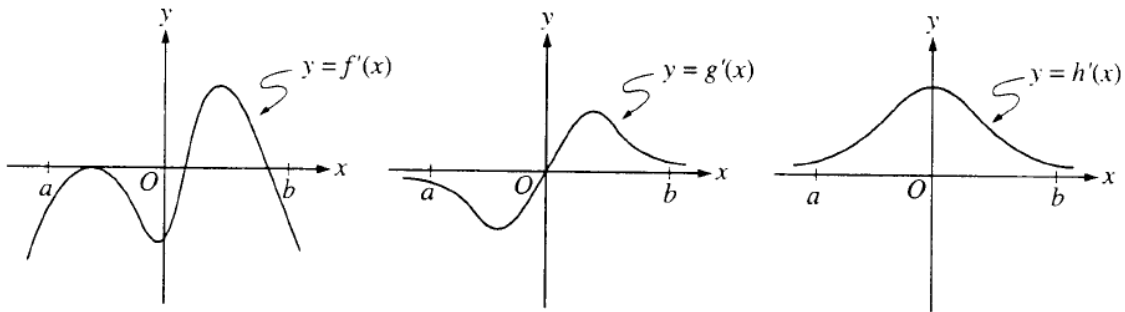
- (A) -0.701
- (B) -0.567
- (C) -0.391
- (D) -0.302
- (E) -0.258

30.

The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C , what is the rate of change of the area of the circle, in square centimeters per second?

- (A) $-(0.2)\pi C$
- (B) $-(0.1)C$
- (C) $-\frac{(0.1)C}{2\pi}$
- (D) $(0.1)^2 C$
- (E) $(0.1)^2 \pi C$

31.



The graphs of the derivatives of the functions f , g , and h are shown above. Which of the functions f , g , or h have a relative maximum on the open interval $a < x < b$?

- (A) f only
- (B) g only
- (C) h only
- (D) f and g only
- (E) f , g , and h

32.

The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

- (A) One
- (B) Three
- (C) Four
- (D) Five
- (E) Seven

33.

Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

- I. f is continuous at $x = 0$.
- II. f is differentiable at $x = 0$.
- III. f has an absolute minimum at $x = 0$.

- (A) I only (B) II only (C) III only (D) I and III only (E) II and III only

34.

If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x , then $\int_1^3 f(2x) dx =$

- (A) $2F(3) - 2F(1)$
- (B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
- (C) $2F(6) - 2F(2)$
- (D) $F(6) - F(2)$
- (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

35.

If $a \neq 0$, then $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$ is

- (A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

36.

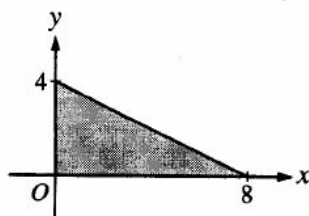
| | | | | |
|--------|----|----|----|----|
| x | 2 | 5 | 7 | 8 |
| $f(x)$ | 10 | 30 | 40 | 20 |

The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of

$$\int_2^8 f(x) dx?$$

- (A) 110 (B) 130 (C) 160 (D) 190 (E) 210

37.



The base of a solid is a region in the first quadrant bounded by the x -axis, the y -axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x -axis are semicircles, what is the volume of the solid?

- (A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

38.

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

- (A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

39.

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

- (A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

40.

If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if

$f'(x) = (x^2 - 4)g(x)$, which of the following is true?

- (A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
- (B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
- (C) f has relative minima at $x = -2$ and at $x = 2$.
- (D) f has relative maxima at $x = -2$ and at $x = 2$.
- (E) It cannot be determined if f has any relative extrema.

41.

If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

- (A) A is always increasing.
- (B) A is always decreasing.
- (C) A is decreasing only when $b < h$.
- (D) A is decreasing only when $b > h$.
- (E) A remains constant.

42.

Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

- I. f has at least 2 zeros.
 - II. The graph of f has at least one horizontal tangent.
 - III. For some c , $2 < c < 5$, $f(c) = 3$.
- (A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

43.

If $0 \leq k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

- (A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436